# Mechanical and Engineering Institute of Aix-Marseille University

## Post-Doctoral position offer

### Research project and job description

**Title**: Hop, Hop, Hopf: Localization, continuation of periodic branches, linear stability analyses at Hopf bifurcation points and in their neighbourhood, for (very) large DAE systems.

**Summary**: Nonlinear problems are characterized by a possible variability in the existence of their solutions in certain ranges of values of the control parameter. In the range of existence of solutions, there may also exist a variability in the number of these solutions. In this last case, it becomes then appropriate to study the stability of these solutions, to be able to predict their temporal persistence in real operating conditions. Indeed, the Hopf bifurcations correspond to the solution points for which there is an intersection of the branches of stationary and periodic solutions. The linear stability of the solution thus changes on both sides of a Hopf bifurcation point. A potential scenario is that the stationary solution is stable (unstable) for a value of the control parameter lower (higher) than the critical value at the Hopf bifurcation point, and that beyond this critical value, it is the periodic solution that is stable (unstable). This is called a supercritical (subcritical) Hopf bifurcation. These stability changes, if not anticipated, can lead to critical operating conditions of the system or process, sometimes leading to accidents or disasters. For example, the vibrational dynamics leading to the coupling of resonance modes by fluid-structure couplings, such as those which caused the Sayano-Shushenskaya hydroelectric power plant disaster in 2009, 74 deaths, [1]), or the catastrophic drying of nuclear reactor core fuel rods by liquid-vapor phase change, [2]), etc.

The nonlinear problems we work on are modeled by algebraic-differential dynamical systems (ADS), which are partly composed of differential equations (ordinary ODE or partial differential equations EDP), while the other part is composed of algebraic equations. The difficulties associated with the numerical resolution of this type of dynamical systems are, on the one hand, a very high numerical stiffness for their temporal integration. On the other hand, if one seeks to determine the stability of an equilibrium solution of such a DAE system, the resolution of a generalized eigenvalue problem must be performed. In its direct formulation this system is singular, which results in as many infinite eigenvalues as there are algebraic equations in the DAE, which characterizes the DAE index. However, when the linearized operator associated to the algebraic part of the system is of maximal rank, one can transform the eigenvalue problem by searching for the eigenfunctions in the kernel space of this operator and thus end up solving a nonsingular eigenvalue problem, provided that the system is "strangeness-free" [3].

Developed at LMA for more than twenty years, the Numerical Asymptotic Method (NAM) is a continuation method which allows to compute stationary solutions of nonlinear problems as a function of control parameters [4]. It consists in expanding all the unknowns of the problem into Taylor series of arbitrary high order, which then allows to transform the original nonlinear system into a series of linear systems, which have the same operator. Compared to classical continuation algorithms of the predictor-corrector type, the added value of this approach consists in exploiting the information contained in the high order Taylor series [5, 6]. The transposition of this know-how to Hopf bifurcations is a real challenge, which will represent an important advance for the treatment of the nonlinear systems on which we work.

**Job description**: The scientific objectives of this work are twofold. In problems governed by (very) large AEDs for which each linear stability analysis represents a significant computational cost. In these cases, one cannot afford to perform a large number of such linear stability analyses. Therefore,

it becomes central to develop a robust algorithm for the localization of Hopf bifurcations in the framework of a continuation by exploiting the information contained in the Taylor series associated with the complete dynamical system. Thus, once an accurate location of the Hopf bifurcation is performed, three linear stability analyses will allow to decide on the subcritical or supercritical nature of the bifurcation. Moreover, local reduction implementations of the Liapunov-Schmidt type, allowing to apprehend the stability of the periodic solution resulting from the Hopf bifurcation [7], can also be considered. This technique of local analysis of dynamical systems, which has been applied to chemical reactors [8], can be generalized to DAE systems.

The implementation of such algorithms in the MANLAB 4 software, developed at the LMA [9, 10] is the first part of this post-doc contract. The second part will consist in their thorough validation on referenced test cases.

**Desired skills**: The skills sought are related to dynamical systems, to their numerical solution, to the treatment of singularities associated with bifurcation points, etc. A good ability to work in a team, but also autonomy, curiosity and scientific rigor will also be assets.

**Period**: from September-October 2022 to August-September 2023 (12 month, with a possibility of 6 months extension)

**Gross salary**: from 2466 € to 2891 €/month, depending on qualification and experience.

**Location**: Marseille, France

**How to apply:** Send your application including:
- A detailed CV with a list of publications;
- A cover letter that contains motivations for this position, along with a list of scientific personalities able to comment on the application.

to:
    Project leader: Marc Medale (marc.medale@univ-amu.fr)
    Project co-leaders: Bruno Cochelin (bruno.cochelin@centrale-marseille.fr),
                Uwe Ehrenstein (uwe.ehrenstein@univ-amu.fr)

**References**:

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[4] B. Cochelin, N. Damil, M. Potier-Ferry, Méthode Asymptotique Numérique, Hermès – Lavoisier (2007).
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[9] ManLab: An interactive path-following and bifurcation analysis software, https:\\manlab.lma.cnrs-mrs.fr
[10] L. Guillot, C. Vergez, B. Cochelin, A Taylor series-based continuation method for solution of dynamical systems, Nonlinear dynamics, 55 (2019).